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The Relationship of the Slope of the Heart Rate Graph Regression with Linear and Nonlinear Heart Rate Dynamics in Stationary Short-time Series

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Abstract—The relationship of the slope of the heart rate graph regression curve (b_1) with periodic (linear) and nonlinear heart rate dynamics has been studied in stationary short-time series (256 points). For estimating nonlinear dynamics, a parameter derived from correlation dimension has been used, which has made it possible to estimate chaotic processes in short-time series. According to the results of the study, the heart rate dynamics in short-time series may be represented as a sum of linear (periodic) and nonlinear (stochastic) processes. The relationships of b_1 with both the linear and the nonlinear heart rate dynamics have been demonstrated. Equations for calculating the absolute and relative (to the periodic oscillation amplitude) noises in the heart rate dynamics in short-time series are proposed. Stochastic nonlinear dynamics in different physiological states of humans have been compared. It has been found that the increase in the relative noise intensity in the heart rate dynamics with an increase in respiration rate is determined not only by the decrease in the amplitude of respiratory waves, but also by an increase in the amplitude of the noise itself. The absolute noise intensity is decreased in the states of neurotic excitement, fatigue, and, especially, mental stress. In the state of rest, nonlinear (stochastic) processes dominate over linear (periodic) ones.

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Key words: nonlinear heart rate dynamics, stochastic processes, correlation dimension, heart rate graph, functional states

INTRODUCTION

When developing the classification of human functional states based on a three-factor model of heart rate variability (HRV) [1], we used the slope of the regression curve of the heart rate graph (b_1). It was assumed that the parameter b_1, which was positively correlated with the heart rate oscillation period, could also serve as an index showing how disordered (chaotic) the heart rate dynamic is. The results of studies aimed at testing this hypothesis are described here.

We considered heart rate as a complex dynamic system exhibiting linear (oscillations at various frequencies) and nonlinear dynamics in stationary short-time series (the sample size was 256 points). Nonlinear dynamics is divided into chaotic and stochastic processes. In the case of chaotic processes, the position of the object at any moment of time can be determined with the use of a mathematical model, provided the initial conditions are known precisely. Therefore, they are referred to as determinate-chaos systems or chaotic systems for short. They are highly sensitive to predetermined initial conditions, which can be set only at a finite accuracy both in physical experiments and in computer simulation. Therefore, long-term prediction of the behavior of chaotic systems is impossible.

To visualize the process, the state of the dynamic system and degree of its organization, a phase space is used. The variables of the state of the dynamic system

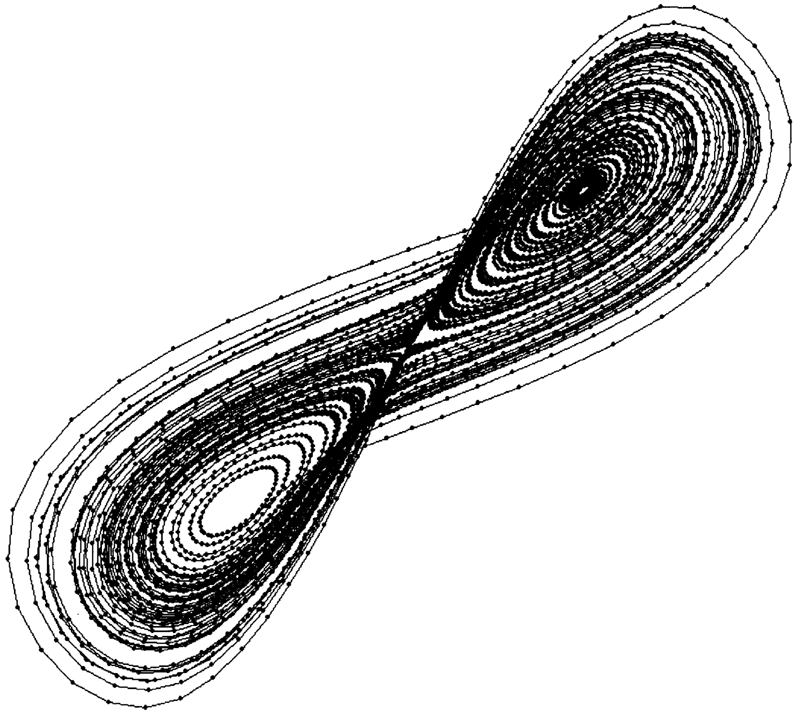

Fig. 1. A computer model of the Lorenz system strange attractor for one variable in a two-dimensional embedding pseudo-phase space ($x(t)$, $x(t + 5)$).

or components of the vector of its state (e.g., the coordinates and velocity of a body) serve as the coordinates of this multidimensional space. The state of the system is shown as a point in the phase space, and the change in the state with time is shown as the movement of the point along the line termed the phase trajectory. The set of points or the subspace in the phase space that the system's phase trajectories approach (to which they are "attracted") after transitional processes fade is termed an attractor. In the case of linear dynamics, an attractor in the form of a point (the limit stabilization of the heart rate, when HRV, if any, is minimal) corresponds to an equilibrium state in the phase space, and closed phase curves (which are observed in heart graphs during controlled breathing tests with a set frequency [2]) correspond to a periodic process (or the limit cycle). Points are never repeated and trajectories never intersect with one another in the phase space of chaotic systems. However, both points and trajectories remain within the attractor, a region of the phase space. These attractors have been termed strange (or chaotic). In a three-dimensional phase space, a strange attractor appears as a set of an infinite number of layers or parallel planes, the distances between some of them approaching infinitely small values [3]. One of the main characteristics of a strange attractor is the sensitivity of its trajectories to initial values. This means that two trajectories that are close to each other in the phase space at a certain initial moment of time exponentially diverge within a short mean time. On the other hand, any attractor has limit sizes; therefore, the exponential divergence of two trajectories cannot be infinite. Sooner or later, the trajectories will approach each other again and remain adjacent for some time or even coincide. The latter, however, is almost improbable (the coincidence of trajectories is a rule for the behavior of linear dynamic systems). The exponential divergence/convergence (also called extension and folding, respectively) of the phase trajectories of the system may be estimated with the use of Lyapunov's indices. To identify nonlinear dynamic processes, it is sufficient to calculate the largest Lyapunov's index ($_1$), which shows whether neighboring phase trajectories, on average, diverge ($_1 > 0$, unstable movement) or converge ($_1 < 0$, stable, regular movement) [3]. If, in the former case, the movement of the dynamic system is confined to a region of the phase space (the attractor), the movement is chaotic (a case of determinate chaos); if it fills the entire phase space, this is a stochastic process. Another quantitative parameter of attractors is the fractal dimension (D), a quantitative characteristic of the set of points in the phase space that shows how densely the points fill the subspace when their number becomes large [3]. The fractal dimensions of attractors are integer for linear processes and fractional for strange attractors. The fractal dimension can be evaluated using the correlation dimension index (D_2): $D_2 = 0$ in the equilibrium state and $D_2 = 1$ during periodic movement. For example, the correlation dimensions of strange attractors of the Henon map and Lorenz system are $D_2 = 1.21$ and $D_2 = 2.05$, respectively [4]. To determine the correlation dimension, the continuous trajectory is subjected to discretization; i.e., it is replaced by a set of N points $\{X_i\}$ in the phase space. Then, the correlation integral ($C(r)$) is calculated. It is equal to the probability that the distance between point pairs ($|X_i - X_j|$) in the phase space is smaller than r. If r values are small, the correlation integral increases with r according to the power law. The correlation dimension equals the corresponding mathematical power, which can be calculated from the slope of the straight line in the ($\log C$, $\log r$) plot.

The method for evaluating the fractal dimension described above requires that the dimension of the phase space (the number of variables necessary for determining the state of the system) should be known and all variables of state could be measured. This is seldom met under actual experimental conditions, where it is possible to trace and measure the evolution of only one variable of state with time. Takens [5] suggested a solution to this problem and Grassberger and Procaccia [4] developed it. The main idea was the following [3]. If the dimension of the phase space is unknown, one cannot know how many variables ($x(t)$, $y(t)$, $z(t)$, ...) are to be measured. Instead, a pseudo-phase space (or embedding space) is constructed with the use of the values of one variable taken at intervals, e.g., ($x(t)$, $x(t + T)$, $x(t + 2T)$, ...).

Figure 1 shows an example of the computer simulation of the strange attractor of the Lorenz system based on one variable in a two-dimensional embedding pseudo-phase space. To determine the correlation dimension, we use sampling measurement $x(t)$ to construct an embedding space with a constantly increasing dimension (m) until the index $D_2(m)$ reaches its asymptotic value ("saturation"), which is taken to be equal to D_2.

Figure 2 shows the plots of correlation dimension D_2 as a function of the dimension of the

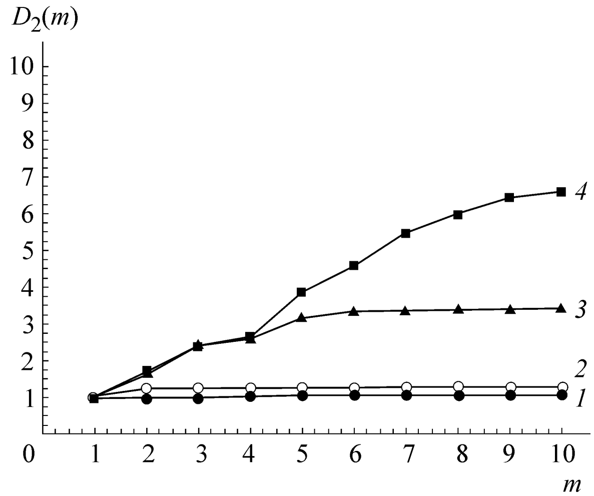

Fig. 2. Correlation dimension ($D_2(m)$) as a function of the dimension of the embedding pseudo-phase space (m) for different models of the time series: (*1*) harmonic oscillations; (*2*) Henon map; (*3*) harmonic oscillations + Gaussian noise; (*4*) Gaussian noise.

embedding pseudo-phase space m ($D_2(m)$) for different models of time series. As expected, the correlation dimension obtained by constructing an embedding pseudo-phase space is 1.0 in the case of harmonic oscillations (periodic movement), and $D_2 = 1.21$ for the strange attractor (the Henon map, determinate chaos). If a Gaussian noise (a stochastic process) is simulated, the increase in embedding space dimension does not lead to the "saturation" of the correlation dimension index (Fig. 2). Numerous studies yielded similar results [6–9]. This is so because a stochastic system (random noise) is described by a (theoretically) infinite number of independent variables, and an increase in the dimension of the embedding space (m) leads to a monotonic increase in the $D_2(m)$ index. If real time series are analyzed, the observed pattern indicates that either a stochastic process occurs or the state of the system is determined by a larger number of parameters. To draw a more definite conclusion, longer time series should be used in this case. The addition of noise to harmonic oscillations leads to an increase in the correlation dimension, but the effect of $D_2(m)$ "saturation" with increasing the embedding pseudo-phase space is retained (Fig. 2). These relationships are analyzed in detail in [9].

In our studies on heart rate dynamics, a substantial restriction is imposed on the use of the correlation dimension parameter–the time series is too small (256 points) for estimating D_2. According to [10], for calculating D_2 by the method of embedding pseudo-

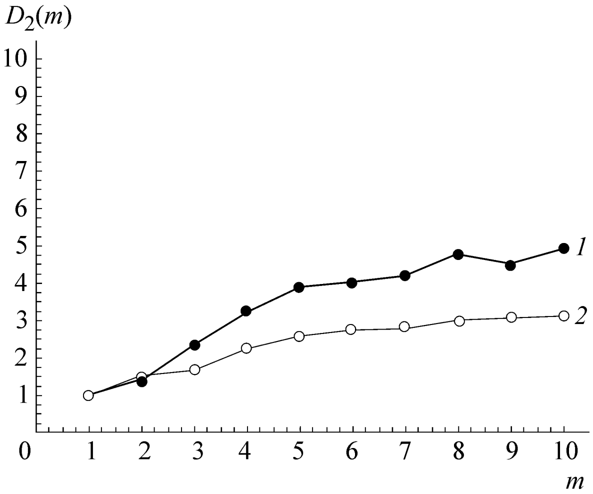

Fig. 3. Correlation dimension ($D_2(m)$) as a function of the dimension of the embedding pseudo-phase space (m) of the heart rate time series for different functional states: (*1*) normal state at rest; (*2*) neurotic excitement.

phase space, the size of the time series should be $10^{2 + 0.4D}$, where D is the fractal dimension of the attractor. The presence of noise in the heart rate leads to an increase in the fractal dimension of the attractor and, hence, in the size of the time series required for precise estimation of the correlation dimension. However, preliminary studies demonstrated that different functional states of humans are characterized by different deviations of the plot of the correlation dimension $D_2(m)$ versus the dimension of the embedding pseudo-phase space (m) from the diagonal (m, m). For example, the more relaxed is the subject, the closer the correlation dimension plot approaches the diagonal (Fig. 3, curve *1*). If the subject is in neurotic excitement, the plot of the correlation dimension farther diverges from the diagonal (Fig. 3, curve *2*). Having analyzed preliminary data, we chose to estimate the intensity of the stochastic process in heart rate dynamics on the basis of the mean sum of squared deviations of the D_2(m vs. m plot from the diagonal:

$$sD_2 \qquad \frac{[D_2(m) \quad m]^2}{N}, \qquad (1)$$

where N is the maximum embedding dimension used for estimating D_2.

If our hypothesis is true, and the parameter b_1 reflects the degree of chaos (indeterminateness) in heart rate, then a positive association must exist between b_1 and sD_2. To test our hypothesis, however, first we had to determine whether it is possible to use the sD_2 parameter for estimating the intensity of noise (a stochastic process) in the heart rate dynamics in short-time series.

EXPERIMENTAL

Heart rate records were obtained in the course of examination of nuclear power station personnel in the Laboratory of Psychophysiological Support of the Novovoronezh Training Center for Nuclear Power Station Personnel (LPPS NTC). The recording of normal cardiac sinus cycles of the electrocardiogram and the subsequent isolation of R–R intervals (in milliseconds) were performed by means of the RITMON-1 and Varikard-1.51 three-channel software –hardware complexes (discretization frequency, 500 Hz). The MABP.DBase-HRV software developed in the LPPS NTC was used to store R–R intervals, edit them (correct artifacts and extrasystoles in the rhythmogram), and calculate the parameters of HRV. Averaged parameters of “sliding” stationary samples of 256 R–R intervals with a step of 10 R–R intervals were used for analysis. The nonparametric Wald–Wolfowitz method [11] was used to confirm that the samples were stationary. The Statistica for Windows 6.0 software was used for statistical analysis.

The results of the examination were used to form a reference group (RG) of 231 healthy men (mean age, 34.0 years; standard deviation (*SD*), 7.8 years), which was confirmed to be stationary. The heart rate was recorded for 10 min in the morning, the subject being in a sitting position. In addition, four “functional” groups of 32 men each were formed on the basis of this examination: FG1, the normal state (age, 32.1 years; *SD* = 5.8); FG2, mental stress during an attention test (age, 28.4 years; *SD* = 4.8); FG4, neurotic excitement in subjects waiting for the attention test to begin (age, 35.2 years; *SD* = 7.6); and FG4, fatigue and decline in functional reserves (age, 31.1; *SD* = .8). The heart rate was always recorded in the morning; the subjects were in a sitting position during the procedure. The duration of recording was 10 min in groups FG1, FG4 (in the resting state), and FG3 (subjects awaiting the start of a psychological test); for FG2 (in the course of psychological testing), the duration depended on the duration of the test. The third task (switching attention) of the Schulte–Gorbov test [12] was used as a mental load (a computerized variant of this method was specially developed in the LPPS NTC). FG3 was characterized by increased neurotic scores on MMPI scales (F.B. Berezin’s variant), difficulty in sitting quietly and motionlessly before the Schulte–Gorbov test, and general excitement and disorganization during the psychophysiological examination. Asthenia (FG4) was diagnosed in the cases of decreased productivity, impaired memory and attention during psychophysiological testing, and complaints of fatigue. All subjects that had no neurotic or asthenic symptoms were assigned to the normal group (FG1).

Two groups were formed according to the results of controlled breathing tests. The first group (CBG1) comprised the results of tests performed in the LPPS NTC. The tests were performed in the morning, the subjects being in the state of rest in a sitting position. The sample of subjects consisted of 58 men (age, 32.4 years; $SD = 4.5$). The subjects evenly breathed for 5 min at a respiration rate of 0.1 Hz (the rate was monitored using a stopwatch). The second group (CBG2) comprised data on seven volunteers (PhysioBank Archives) who breathed at a rate of 0.25 Hz in a lying position for 10 min, the respiration rate being set with a metronome (http://www.physionet.org/physiobank/database/meditation/data/metron/). The selected data were stationary and displayed a distinct spectral peak at the respiration rate of controlled breathing.

The MABP-Chaos software was developed in the LPPS NTC specially for analyzing the nonlinear heart rate dynamics. This software permits simulation of a wide spectrum of linear and nonlinear processes and calculation of parameters for both model data and actual heart rate time series. To calculate the correlation integral (delay, 1; window size, 80), we used the algorithm proposed by M.T. Rosenstein (http://www.physionet.org/physiotools/lyapunov/11d2/1d2.c). The correlation dimension was calculated as the mean slope of the plot (log C, log r). In addition, Rosenstein's algorithm was used to calculate the highest Lyapunov's exponent ($_1$). Rosenstein's study [13] presents a detailed substantiation of the use of this algorithm for estimating $_1$ in short-time series. The sD_2 value was calculated by Eq. (1) at $N = 10$.

Regarding HRV parameters, we used the standard deviation of normal (N–N) intervals (*SDNN*) (N–N intervals are the R–R intervals between QRS complexes of normal sinus cycles without artifacts or extrasystoles) and the slope (b_1) of the regression curve of the heart rate graph (N–N_n, N–N_{n+1}) [1]. The heart rate graph is an example of a two-dimensional embedding pseudo-phase space plotted for the time series of N–N intervals.

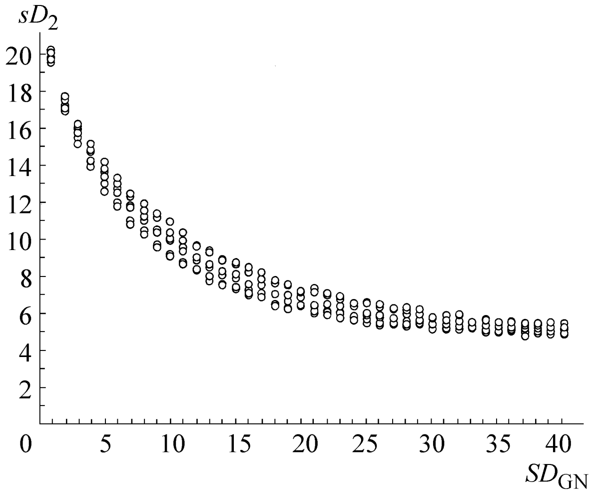

Fig. 4. The sD_2 index at different levels of "Gaussian noise" (SD_{GN}) in harmonic oscillations ($Am = 20$).

We used the model of superimposing "Gaussian noise" (GN) on harmonic oscillations (such use substantiated in [14]). Harmonic oscillations with superimposed noise were simulated using the formula

$$Am\cos(2\ f) + Am\sin(2\ f) + M + RandG(0,\ SD_{GN}),$$

where Am is the wave amplitude, f is the oscillation frequency, M is the mean value, and *RandG* is a function from the Delphi-5 package for generating GN (0 is the mean value and SD_{GN} is the standard deviation). The period of the harmonic oscillations ($1/f$) varied from 3 to 256, SD_{GN} varied from 1 to 40, $Am = 20$. For each level of GN (SD_{GN}), we simulated 100 samples, which were used to calculate averaged parameters to be used in subsequent analysis.

RESULTS AND DISCUSSION

As expected, the simulation of harmonic oscillations with superimposed GN demonstrated an inverse dependence of sD_2 on the GN level (Fig. 4): the higher the noise level in the signal (SD_{GN}), the closer the correlation dimensions to the diagonal of the $D_2(m)$ plot. Nonlinear regression analysis yielded the following dependence of sD_2 on the noise level: $sD_2 = -30 + 51(SD_{GN})^{-0.1}$. This relationship accounted for 98.23% of the variance ($R = 0.9911$, $p < 0.001$). It is important that sD_2 was not related to the frequency of the harmonic oscillations. In addition, we analyzed the relationship of sD_2 with the oscillation amplitude.

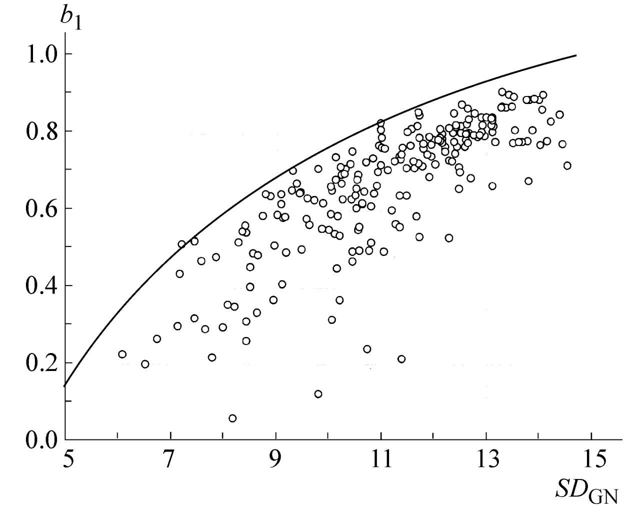

Fig. 5. The slope of the heart rate graph regression curve (b_1) relative to the sD_2 for the reference group. The solid line shows the dependence of b_1 on sD_2 calculated for harmonic oscillations at different levels of "Gaussian noise" and a period of 256 (Am = 20).

It was assumed that sD_2 reflected the ratio of the noise to the oscillation amplitude ($nSD_{GN}\% = 100SD_{GN}/Am$), rather than its absolute value. Then, according to the results of nonlinear regression analysis, the relative level of GN could be expressed as

$$nSD_{GN}\% \quad 100 \; \frac{18.665}{sD_2 \quad 11.771}^{\,5.320}. \qquad (2)$$

This relationship accounted for 91.60% of the variance ($R = 0.9572$, $p < 0.001$). To confirm the hypothesis that sD_2 was related to the relative noise level, we simulated the following harmonic oscillations: the amplitude varied from 2 to 20, SD_{GN} varied from 1 to 40 (the data were averaged over 100 samples), and the oscillation period had a fixed value of 16. The following linear relationship between the starting ($nSD_{GNs}\%$ 150) and calculated ($nSD_{GNc}\%$) relative levels of GN was obtained: $nSD_{GNs}\% = 0.5126 + 1.0925nSD_{GNc}\%$ ($R = 0.9750$, $p < 0.001$). This showed that the values of the studied parameters significantly coincided and confirmed the hypothesis on the relationship between sD_2 and the GN relative to the amplitude of the original signal. Similar results were obtained when harmonic oscillations with periods of 8 and 32 were simulated.

Analysis of the b_1 parameter calculated for the graphs of harmonic oscillations with superimposed GN confirmed the complex nonlinear dependence of the slope of the graph regression curve on both the oscillation frequency and the noise level (the relationship was negative in both cases). A simplified model of linear regression analysis yielded the following Beta coefficients (reflecting the relative contributions of independent variables to the prediction of the dependent variable): Beta = –0.60; for SD_{GN}, Beta = –0.72 ($R = 0.941$, $p < 0.001$). The dependence of b_1 on sD_2 may be presented in the following general form: $b_1 = P_0 + P_1 sD_2^{P_3}$ (the equation coefficients P_0, P_1, and P_3 depending on the harmonic oscillation frequency). For an oscillation period of 256, nonlinear regression analysis yielded the following equation: $b_1 = 1.80 - 4.971sD_2{-0.675}$. This relationship accounted for 99.53% of the variance ($R = 0.0076$, $p < 0.001$).

For parameters b_1 and sD_2 calculated from the results of heart rate recording in the reference group (Fig. 5), we obtained a significant relationship: $r = 0.789$, $p < 0.001$ (b_1 increased with an increase in sD_2 reflecting the relative noise level in the original signal; however, the dependence was nonlinear). As evident from Fig. 5, all values of b_1 as a function of sD_2 fell either below the curve calculated for harmonic oscillations with superimposed GN and a period of 256 (the "limit" period of the wave discernible by spectral analysis in a sample of 256 runs) or approximately on this curve. Note that this relationship was also found for groups of subjects in different functional states and subjects performing controlled-breathing tests. In our opinion, this is additional evidence that linear (periodic) and nonlinear (stochastic) dynamics form the basis of heart rate dynamics in short-time series. The "limit curve" is the boundary for all points of the plot of b_1 as a function of sD_2 at various oscillation periods and noise levels. The table shows the mean values and standard deviations of the parameters of HRV nonlinear dynamics for the reference group (RG). Note that the values of the highest Lyapunov's exponent ($_1$) were higher than zero for all groups studied. This indicates that the dynamic system (cardiac rhythm) is in an unstable state, which may be related to either determinate chaos or stochastic process.

Now let us consider controlled breathing tests and compare the obtained results (table) for two groups (CBG1 and CBG2) where the subjects breathed at rate of 0.1 and 0.25 Hz, respectively. In the spectral density plot, CBG1 was characterized by a peak at a frequency of 0.099 Hz ($SD = 0.005$); and CBG2, by a

Nonlinear dynamics and HRV parameters (the mean and *SD*) for different groups

Parameter	$_{1}$	b_1	sD_2	nSD_{GN}%	SD_{GN}	*SDNN*
RG	0.147 (0.023)	0.665 (0.180)	11.14 (1.880)	38.00 (19.32)	16.54 (11.64)	47.98 (21.72)
CBG1	0.150 (0.021)	0.832 (0.060)	14.35 (1.295)	17.46 (5.22)	11.43 (3.26)	66.84 (7.15)
CBG2	0.116 (0.009)	0.247 (0.247)	8.89 (1.485)	62.91 (24.44)	29.24 (9.14)	59.07 (17.96)
FG1	0.135 (0.014)	0.368 (0.192)	8.28 (1.374)	75.85 (36.85)	29.24 (11.05)	51.46 (12.49)
FG2	0.141 (0.025)	0.746 (0.117)	11.60 (2.470)	35.59 (18.40)	4.18 (1.99)	13.11 (2.94)
FG3	0.185 (0.020)	0.872 (0.071)	14.33 (1.430)	17.73 (5.85)	11.11 (3.71)	65.44 (18.69)
FG4	0.133 (0.015)	0.520 (0.103)	8.86 (1.435)	63.31 (22.32)	11.10 (3.08)	21.56 (3.04)

peak at 0.232 Hz (*SD* = 0.020). To calculate spectral parameters without preliminarily transforming the original set of R–R intervals into an equidistant series, we corrected the frequency peaks on the basis of the average R–R interval [15]. The coefficient of correlation between b_1 and sD_2 was 0.839 ($p < 0.001$); however, the dependence remained nonlinear. To test the hypothesis on the differences between parameters for CBG1 and CBG2, we used the nonparametric Mann–Whitney U test and Kolmogorov–Smirnov test for unrelated samples. The two groups significantly differed ($p < 0.001$) in all parameters except *SDNN*. The relative noise level (nSD_{GN}%) considerably increased with increasing respiration rate (from 17.46 to 62.01% at rates of 0.1 and 0.25 Hz, respectively).

An important question arises as to how largely the changes in nSD_{GN}% were determined by the noise level, rather than the amplitude of heart rate oscillations. Earlier [16], it was found that the amplitude of respiratory waves estimated by means of spectral analysis increased as the respiration rate decreased. Our results confirm this finding. We estimated the maximum spectral density of respiratory peaks at different respiration rates. The mean maximum spectral densities of respiratory peaks in CBG1 and CBG2 were 110 622 ms^2 (*SD* = 28 448) and 296 959 ms^2 (*SD* = 54 042), respectively. Both U and tests showed a significant difference between these values ($p < 0.001$). To estimate the absolute noise levels in heart rate dynamics, we used the following simplified formula: $SDNN^2 = SD_{GN}^2 + Am^2$. The total oscillation amplitude can be calculated as $Am = SD_{GN}/nSD_{GN}$. As a result, we obtain the following equation for the absolute GN level:

$$SD_{GN} \quad \frac{SDNN nSD_{GN}}{(nSD_{GN}^2 \quad 1)^{0,5}}. \qquad (3)$$

The table shows the mean GN levels (in standard deviation units, ms) for different groups. CBG1 and CBG2 significantly differ in SD_{GN} according to the U and tests ($p < 0.001$). The mean proportion of SD_{GN} in the total HRV ($100SD_{GN}/SDNN$) was 49.50% for the subjects that breathed at a rate of 0.25 Hz (CBG2) and 17.10% for CBG1. Thus, the results confirmed that the increase in the relative noise level with an increase in respiration rate was accounted for not only by a decrease in the amplitude of respiratory waves, but also by an increase in amplitude of the noise itself. According to our data, controlled breathing at a rate of 0.1 Hz is characterized by a significantly higher b_1 (according to the U and tests, $p < 0.001$), which is determined by not only a lower respiration rate, but also a lower intensity of stochastic processes (noise) in the heart rate dynamics. The increase in the noise level against the background of a decreasing respiratory wave amplitude with an increase in respiration rate explains why the total HRV (*SDNN*) changes only slightly.

In conclusion, let us consider the results of the comparison between the four functional groups with respect to the nonlinear dynamics and HRV (table). FG1 (the normal state) and FG4 (fatigue) did not differ significantly from each other in the relative noise level parameters (sD_2 and nSD_{GN}%) according to the U or tests ($p < 0.11$). As in the case of controlled breathing tests, we estimated the absolute noise level in the heart rate dynamics for the functional states studied, which allowed us to determine the patterns of stochastic processes in the state of fatigue more accurately. As the working capacity decreases and a subject becomes to feel tired, the tone of the vagus nerve estimated by SD_{GN} is significantly decreased ($p < 0.001$ for both U and tests). This is confirmed by the conclusions of our earlier studies [17], where the total HRV was decreased (which is characteristic of fatigue), while the relative activity of the parasympathetic nervous system (as estimated by nSD_{GN}%) remained unchanged, against the background of a general decrease in the autonomic nervous system tone (the proportion of SD_{GN} in the total HRV in the normal state at rest and during fatigue was 56.82 and 51.48%, respectively). Note the absence of significant differences between CBG2, FG1, and FG4 with respect to the relative noise level (as well as the SD_{GN} to *SDNN* ratio), while the *SDNN* and SD_{GN} were considerably decreased in the state of fatigue ($p < 0.001$ for both U and tests). The significant difference between FG1 and FG4 in b_1 ($p < 0.001$ for the U test and $p < 0.025$ for the test) show that, whereas the difference in the relative noise level in heart rate dynamics was small, fatigue was accompanied by an increase in the oscillations at lower frequencies (which was reflected by the increase in b_1). The b_1 values were the lowest in CBG2: breathing control at a fixed frequency (0.035 Hz) determined not only a high relative noise level, but also a complete domination of high-frequency oscillations in the heart rate. FG3 (stress) and FG1 (the normal state) significantly differed from each other in all parameters ($p < 0.001$ for both U and tests). Mental concentration on a problem may both decrease the relative noise level in heart rate dynamics (a decrease in the vagus tone) and increase the activity of the sympathetic nervous system, which is expressed in an increase in low-frequency oscillations. In our study, the strongest effect of this process was a decrease in absolute noise level, which confirmed that the vagus nerve had a weak effect in the state of a high mental load (the proportion of SD_{GN} in the total HRV in stress was 31.88%). All this is reflected in the increase in b_1 and a decrease in the total HRV. During neurotic excitement (FG2), the relative noise level was significantly lower than during mental stress, the total HRV remaining high. The estimate of the absolute noise level confirmed that the vagus tone was decreased during neurotic excitement, and the high b_1 values indicated that the increase in the total HRV was primarily determined by an increase in the amplitude of low-frequency oscillations (the proportion of SD_{GN} in the total HRV in the state of neurotic excitement was 16.98%). The data on the neurotic excitement group did not differ significantly from the results of the test of controlled breathing at a rate of 0.1 Hz ($p > 0.1$ for both U and tests). Only b_1 was significantly higher ($p < 0.001$) in FG3, which confirms the presence of waves with a frequency lower than 0.1 Hz in the heart rate dynamics of neurotically excited subjects [17]. There is evidence [18] that nonlinear processes in heart rate dynamics are mainly mediated by the vagus influence. The results of our comparison of four functional groups with respect to the parameters of nonlinear dynamics and HRV confirm this hypothesis. Our data also lead to the conclusion that, contrary to the opinion of some researchers [12], stochastic processes similar to "Gaussian noise," rather than determinate chaos, underlie the nonlinear heart rate dynamics in stationary short-time series.

CONCLUSIONS

(1) The results of studies using simulated data (harmonic oscillations with superimposed GN) and actual time series of heart rate lead to the conclusion that the parameter sD_2 (the mean sum of squared deviations of the plot of correlation dimension ($D_2(m)$) as a function of the dimension of the embedding pseudo-phase space (m) from the diagonal (m, m)) can be used for estimating the intensity of stochastic processes in heart rate dynamics in small samples (256 R–R intervals).

(2) According to the results obtained, sD_2 reflects the ratio of the noise level to the amplitude of heart rate periodic oscillations, rather than the absolute noise level in the original signal. Equations for calculating the relative and absolute noise levels in heart rate dynamics (Eqs. (2) and (3), respectively) have been proposed.

(3) It has been established that the slope of the heart rate graph regression (b_1) reflects both linear

(periodic oscillations) and nonlinear (stochastic noise) processes in heart rate dynamics in stationary short-time series. Two main parameters, the period of heart rate oscillations and the relative noise level, affect the b_1 value. An increase in the oscillation period (which is characteristic of the activities of the sympathetic nervous system and central regions of the cerebral cortex) increases b_1, whereas an increase in the relative noise level (the activity of the vagus nerve) decreases b_1.

(4) Analysis of the absolute and relative noise levels in heart rate dynamics in different functional sates and during controlled breathing tests has demonstrated that the increase in the relative noise level in heart rate dynamics is determined by not only a decrease in the respiratory wave amplitude, but also an increase in the amplitude of the noise itself. The absolute noise level (and, hence, the vagus tone) is decreased in the states of neurotic excitement, fatigue (with the relative noise level remaining unchanged), and, especially, mental stress (concentration on a problem). In the states of fatigue and stress, this is accompanied by a decrease in the total HRV (*SDNN*). In the case of neurotic excitement, an increase in the total HRV is mainly determined by an increase in the low-frequency oscillation amplitude, as evidenced by high b_1 values. This accounts for the marked decrease in the relative noise level in the given state. In the normal state at rest, both relative and absolute noise levels are high (a high tone of the vagus nerve): nonlinear (stochastic) processes dominate over linear ones (periodic oscillations).

(5) The highest Lyapunov's exponent ($_1$) is higher than zero for all groups studied. This indicates an unstable state of the heart rate dynamic system, which may be accounted for by either determinate chaos or a stochastic process. The results of our studies allow us to conclude that the total HRV in stationary short-time series is composed of a periodic component and indeterminate chaos (stochastic noise). The stochastic noise level, which reflects the vagus activity, may make a considerable contribution to the total HRV, as assumed by some authors [20].

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